

## MATHEMATICS

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Answer the following questions and write your answers in the boxes provided.

1) Let  $x = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$  and  $y = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$ . Calculate  $x^2 - y^2$ .

2) Solve the equation  $x^3 - x^2 - 10x - 8 = 0$ .

$x =$

3) Solve the equation  $2 \sin^2 x - \cos x = 1$  ( $0 \leq x < 2\pi$ ).

$x =$

4) Solve the equation  $2^{2x+2} + 3 \cdot 2^x - 1 = 0$ .

$x =$

5) Solve the inequality  $(\log_3 x)^2 < \log_9 x^4$ .

6) Solve the inequality  $\sin 2x > \sqrt{2} \sin x$  ( $0 \leq x < 2\pi$ ).

7) Let  $\vec{a} = (1, 2, 3)$ ,  $\vec{b} = (3, 2, 1)$ ,  $\vec{c} = (5, 4, 3)$ . Find the value of  $t$  such that  $\vec{a} + t\vec{b}$  is parallel to  $\vec{c}$ .

8) Let  $O(0, 0)$  and  $A(3, 1)$ . Let  $A'$  be the symmetric point of  $A$  with respect to the line  $y = 2x$ . Calculate the area of the triangle  $OAA'$ .

9) The sequence  $\{a_n\}$  satisfies the following conditions. Calculate  $\sum_{n=1}^5 (a_n - 5)$ .

$$a_1 = 3, \quad a_{n+1} = 2a_n \quad (n = 1, 2, 3, \dots)$$

10) Calculate  $\lim_{x \rightarrow 0} (\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + x})$ .

11) Let  $f(x) = \frac{\cos x}{\sqrt{e^x}}$ . Calculate  $f'(0)$ .

$$f'(0) =$$

12) Calculate  $\int_1^2 (3x^2 - 4x) \log_e x \, dx$ .

2 Let  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ . Answer the following questions and write your answers in the boxes provided.

1) Let  $A = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$  and  $B = \begin{pmatrix} x & 3 \\ 3 & 6 \end{pmatrix}$ . Find the value of  $x$  which satisfies  $AB = BA$ .

$x =$

2) Let  $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$  and  $B = \begin{pmatrix} -2 & x \\ 4 & y \end{pmatrix}$ . Find the values of  $x$  and  $y$  which satisfy  $BA = O$ .

$x =$                        $y =$

3) Let  $A$  satisfying  $A^2 = A - I$ . Find  $A^{15}$ .

$A^{15} = \begin{pmatrix} & \\ & \end{pmatrix}$

3 Answer the following questions and write your answers in the boxes provided.

1) Calculate  $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$ .

2) Calculate  $\int_0^{\frac{\pi}{4}} \cos^3 x \, dx$ .

3) Calculate  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin x + 2 \cos x)^3 \, dx$ .